

GENERAL PROBLEMS OF HEAT EXCHANGE
IN A MOVING LAYER

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An approximate analytical solution is obtained for the problem of steady heat exchange in a moving layer in the presence of heat and mass sources in the gas stream. A numerical-analytical method is developed for the solution of the problem of nonsteady heat exchange of a layer by convection or radiation with the simultaneous action of different disturbing factors.

Heat exchange in a layer moving with variable velocity represents a general case of layer heat exchange, since a stationary layer can be considered as moving with zero velocity. A method of solving problems of heat exchange in a stationary layer based on the use of a general solution of the equation of thermal conduction was examined in [1]. This method is applied below to problems of heat exchange in a moving layer formulated in general form.

First let us examine the steady mode of heat exchange between a layer of massive bodies of the simplest shape and an opposing gas stream in which sources of mass and heat act.

The effect of mass sources is expressed in variation in the flow rate of gas along the length of the layer. The flow rate of the gas and the power of the heat sources are given in the form of arbitrary functions of the time the body stays in the oven or of its coordinate relative to the entrance to the oven. The heat losses are proportional to the average temperature of the gas in the oven. We neglect heat conduction along the layer.

The initial system of equations and the boundary conditions have the following form:

equation of heat conduction

$$\frac{\partial t(r, Fo)}{\partial Fo} = \frac{\partial^2 t(r, Fo)}{\partial r^2} + \frac{v}{r} \cdot \frac{\partial t(r, Fo)}{\partial r}, \quad (1)$$

equation of thermal balance of the gas stream

$$\bar{t}(Fo) = \gamma(Fo) [T(Fo) - 1] + Q(Fo) - \kappa' Fo, \quad (2)$$

boundary conditions

$$\left. \frac{\partial t}{\partial r} \right|_{r=1} = Bi [T(Fo) - t(1, Fo)]; \quad \left. \frac{\partial t}{\partial r} \right|_{r=0} = 0; \quad (3)$$

$$Fo = 0, \quad t = t^0(r), \quad T = 1; \quad (4)$$

$$t = \frac{t_s - t_s^0}{t_g^0 - t_s^0}; \quad T = \frac{t_g - t_s^0}{t_g^0 - t_s^0}; \quad Fo = \frac{a\tau}{R^2}; \quad Bi = \frac{\alpha R}{\lambda};$$

$$r = \frac{y}{R}; \quad \gamma(Fo) = \frac{V_g(Fo) c_g}{V_s c_s}; \quad Q(Fo) = \frac{q(Fo)}{V_s c_s (t_g^0 - t_s^0)};$$

$$\kappa' = \kappa \frac{t_{cal} + t_g^0}{2(t_g^0 - t_s^0)} \cdot \frac{f_{li} R^2 \rho}{\lambda}.$$

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The general solution of Eq. (1) with the condition that the gas temperature is some function of the time is well known [2, 3]

$$t = T - \sum_{n=1}^{\infty} A(\mu_n, r) \exp(-\mu_n^2 Fo) \left[1 + \int_0^{Fo} \frac{dT}{d\omega} \exp(\mu_n^2 \omega) d\omega \right] + \sum_{n=1}^{\infty} A'(\mu_n, r, t^0) \exp(-\mu_n^2 Fo). \quad (5)$$

Since in the majority of cases the processes of heating of the substance in the moving layer are completed when $Fo > 0.5$, in the solution of (5) we confine ourselves to the first terms of the sums. An expression for the average-mass temperature of the body follows from (5):

$$\bar{t} = T - B(\mu) \exp(-\mu^2 Fo) \left[1 + \int_0^{Fo} \frac{dT}{d\omega} \exp(\mu^2 \omega) d\omega \right] + B'(\mu, t^0) \exp(-\mu^2 Fo). \quad (6)$$

The root μ of the characteristic equation and the coefficients A, A', B, and B' are known from the solution of the equation of thermal conduction for a single body with boundary conditions of the third kind and are presented in the literature [2].

Let us differentiate (6) with respect to Fo:

$$\frac{d\bar{t}}{dFo} = \frac{dT}{dFo} + \mu^2 \left\{ B \exp(-\mu^2 Fo) \left[1 + \int_0^{Fo} \frac{dT}{d\omega} \exp(\mu^2 \omega) d\omega \right] - B' \exp(-\mu^2 Fo) \right\} - B \frac{dT}{dFo}. \quad (7)$$

Using (6) we transform Eq. (7) to the form

$$\frac{d\bar{t}}{dFo} = \frac{dT}{dFo} + \mu^2 (T - \bar{t}) - B \frac{dT}{dFo}. \quad (8)$$

Based on the fact that Eq. (8) must satisfy the condition (2), we arrive at a differential equation relative to the gas temperature

$$\frac{dT}{dFo} + M(Fo) T = N(Fo), \quad (9)$$

for which the general integral has the form

$$T = \exp\left(-\int_0^{Fo} M dFo\right) \left[1 + \int_0^{Fo} N \exp\left(\int_0^{Fo} M dFo\right) dFo \right], \quad (10)$$

$$M = \frac{\frac{d\gamma}{dFo} - \mu^2 + \gamma\mu^2}{\gamma - 1 + B}; \quad N = \frac{\frac{d\gamma}{dFo} - \frac{dQ}{dFo} + \gamma\mu^2 - Q\mu^2 + \kappa' + \kappa'\mu^2 Fo}{\gamma - 1 + B}.$$

The average-mass temperature of the body is determined from the condition (2) while the temperature at any point through the thickness can be found from the following equation, obtained through substitution from (6) into (5):

$$t(r, Fo) = T + \frac{A}{B} (\bar{t} - T) - \frac{AB' - A'B}{B} \exp(-\mu^2 Fo). \quad (11)$$

If $t^0 = \text{const}$ in the condition (4) then according to [2] we have $A' = At^0$, $B' = Bt^0$, and the last term in (11) is reduced to zero.

Let us examine a particular case of the solution (10) which is characteristic for continuous ovens with multizone heating, in which the sources of mass and heat are concentrated at the junctions of the zones and can be expressed by step functions:

$$\gamma = \sum_{i=1}^p \gamma_i |1 - \varphi(Fo - Fo_i)|; \quad Q = \sum_{i=1}^{p-1} \gamma_i (T_{in} - 1) \varphi(Fo - Fo_i); \quad (12)$$

here $T_{in} = (t_{cal} - \bar{t}_s^0) / (t_g^0 - \bar{t}_s^0)$, $\gamma_i = (V_{gi} c_{g_i} / V_{sc} c_s)$, and φ is the unit function [4].

In accordance with the property of the unit function

$$\frac{d\gamma}{dFo} = - \sum_{i=1}^p \gamma_i \delta(Fo - Fo_i); \quad \frac{dQ}{dFo} = \sum_{i=1}^{p-1} \gamma_i (T_{in} - 1) \delta(Fo - Fo_i), \quad (13)$$

δ is the Dirac delta function.

After substitution of (12) and (13) into (10) and integration, the following recurrent equation is obtained for the gas temperature at the end of the n -th zone of the oven counted from the loading site of the materials:

$$T_n = T_{n-1} \exp(-S_n) + E_n. \quad (14)$$

With the condition $\sum_{i=n}^p \gamma_i \neq 1$

$$E_n = \exp[-D(Fo_n - Fo_{n-1})] \left[C + \frac{\gamma_{n-1} T_{in}}{1 - \sum_{i=n}^p \gamma_i - B} + \kappa' \frac{DFo_{n-1} - 1}{D(1 - \sum_{i=n}^p \gamma_i)} \right] - C - \kappa' \frac{DFo_n - 1}{D(1 - \sum_{i=n}^p \gamma_i)}; \quad S_n = \frac{\gamma_{n-1}}{1 - \sum_{i=n}^p \gamma_i - B} + DFo_n;$$

$$C = \frac{\sum_{i=1}^p \gamma_i - \sum_{i=1}^{p-1} \gamma_i T_{in} + \frac{\kappa'}{\mu^2}}{1 - \sum_{i=n}^p \gamma_i}; \quad D = \mu^2 \frac{1 - \sum_{i=n}^p \gamma_i}{1 - \sum_{i=n}^p \gamma_i - B}.$$

With $\sum_{i=n}^p \gamma_i = 1$

$$E_n = \frac{\mu^2 \kappa'}{2B} (Fo_n^2 - Fo_{n-1}^2) + \frac{\mu^2 \sum_{i=1}^p \gamma_i - \mu^2 \sum_{i=1}^{n-1} \gamma_i T_{in} - \gamma_{n-1} T_{in}}{B} (Fo_n - Fo_{n-1}); \quad S_n = - \frac{\gamma_{n-1}}{B}.$$

Finally, taking $\gamma = \text{const}$, $Q = 0$, and $\kappa' = 0$ in (10) we obtain a solution for steady counterflow without sources of heat and mass in the gas:

$$T = \frac{\gamma}{\gamma - 1} + \frac{1}{1 - \gamma} \exp\left(\frac{\gamma - 1}{1 - \gamma - B} \mu^2 Fo\right) \quad \text{for } \gamma \neq 1, \quad (15)$$

$$T = 1 + \frac{\mu^2 Fo}{B} \quad \text{for } \gamma = 1.$$

The results of calculations from Eq. (15) and from certain exact solutions [5, 6] almost coincide for $Fo \geq 0.2$.

Let us consider a general case of nonsteady layer heat exchange. Suppose there is an infinitely long layer of massive bodies of the simplest shape (Fig. 1) moving with a variable velocity v . We will neglect heat conduction along the layer. At the starting time of the process the oven is located with the layer at the entrance. It is valid to assume that the layer remains stationary while the oven moves along it in the opposite direction with the given velocity v . The heating of the substance will take place in the section $St_{end} < St < St_{sta}$ of the layer which is in the oven at a given time. The heating is completed in the segment $0 < St < St_{end}$ and has not yet started in the region $St > St_{sta}$. The time during which the oven covers a distance equal to St_{sta} will be called the heating delay time at a given section of the layer (τ_d). If the total

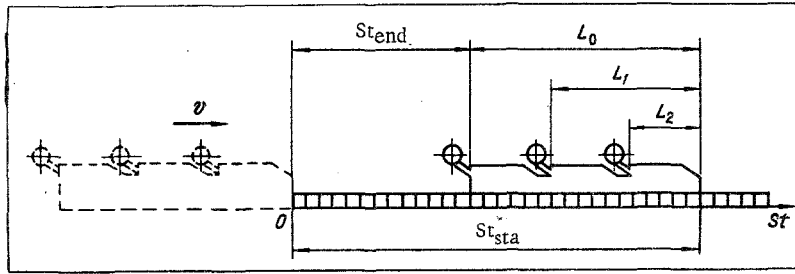


Fig. 1. Model of heating of a moving layer of solid bodies in a pass-through oven.

duration of the process is τ' then the heating time of the substance at a given section of the layer is equal to $\tau = \tau' - \tau_d$. Within the oven the gas moves along the layer with velocity w . The gas velocity relative to the layer is $u = v \pm w$ (the upper sign for counterflow and the lower sign for direct flow). In practice $w \gg v$ and therefore it can be assumed that $u \approx w$ and the flow rate of the gas is determined by its absolute velocity. The rate of movement of the substance and the temperature of the substance and gas at the oven entrance are arbitrarily varied with time. Sources of mass and heat act in the gas. The flow rate of the gas and the power of the heat sources are given in general form as a function of the time and the coordinate along the length of the layer. The initial temperature of the substance is some function of the coordinates through the thickness of the body and along the length of the layer. The heat losses are a fixed function of the gas temperature.

The mathematical formulation of the problem comes down to the following system of equations and boundary conditions:

$$\frac{\partial t(r, Fo, St)}{\partial Fo} = \frac{\partial^2 t(r, Fo, St)}{\partial r^2} + \frac{v}{r} \cdot \frac{\partial t(r, Fo, St)}{\partial r}; \quad (16)$$

$$t(1, Fo, St) - T(Fo, St) = z(Fo', St) \frac{\partial T}{\partial St} + \kappa'(T) - Q'(Fo', St); \quad (17)$$

$$\left. \frac{\partial t}{\partial r} \right|_{r=1} = Bi [T - t(1, Fo, St)]; \quad \left. \frac{\partial t}{\partial r} \right|_{r=0} = 0; \quad (18)$$

$$\left. \begin{aligned} Fo = 0, \quad t = t^0(r, St) \\ \text{for direct flow } St = St_{sta} \quad T = T_{in}(Fo') \\ \text{for counterflow } St = St_{end} \quad T = T_{in}(Fo') \end{aligned} \right\}; \quad (19)$$

$$\left. \begin{aligned} St_{sta} = \int_0^{Fo'} v(Fo') dFo', \quad St_{end} = 0, \quad \text{if } St_{sta} \leq L_0 \\ St_{end} = St_{sta} - L_0, \quad \text{if } St_{sta} > L_0 \end{aligned} \right\}. \quad (20)$$

The dimensionless values in Eqs. (16)-(20) are: $t = t_g/t_{cal}$; $T = t_g/t_{cal}$; $Fo' = at'/R^2$; $Fo = Fo' - a\tau_d/R^2$; $Bi = \alpha R/\lambda$; $St = \alpha f/V_{max}c_g$; $z = V_g(Fo', St)/V_{max}$; $v(Fo') = dSt/dFo'$; $L_0 = \alpha f_{act}/V_{max}c_g$; $Q' = q'(Fo', St)/\alpha t_{cal}$. The general solution of Eq. (16) has the form [1]

$$t(r, Fo, St) = T(Fo, St) - \sum_{n=1}^{\infty} [A(r, \mu_n) G_n - A'(r, \mu_n, t^0)] \exp(-\mu_n^2 Fo), \quad (21)$$

$$G_n = T(0, St) + \int_0^{Fo} \frac{\partial T}{\partial \omega} \exp(\mu_n^2 \omega) d\omega;$$

at the surface of the body

$$t(1, Fo, St) = T(Fo, St) - \sum_{n=1}^{\infty} [A(1, \mu_n) G_n - A'(1, \mu_n, t^0)] \exp(-\mu_n^2 Fo). \quad (22)$$

By requiring that the latter equation satisfy the condition (17) we obtain an equation for the gas temperature in the moving section of heating:

$$z \frac{\partial T}{\partial St} = Q' - \kappa'(T) - \sum_{n=1}^{\infty} [A(1, \mu_n) G_n - A'(1, \mu_n, t^0)] \exp(-\mu_n^2 Fo). \quad (23)$$

The analytical solution of Eq. (23) even for a stationary layer and $z = \text{const}$ [1] is unsuitable for calculations. It is solved most conveniently using an electronic computer.

Let us convert to finite differences with respect to time (ΔFo) and the length of the layer (ΔSt):

$$\text{Fo}'_k = k\Delta\text{Fo}, \text{St}'_m = m\Delta\text{St}, \text{Fo}_{m+1, k} = \text{Fo}'_k - \frac{\alpha\tau_{d, m+1}}{R^2},$$

where $k = 0, 1, 2, \dots$; $m = 0, 1, 2, \dots$;

$$\text{St}_{\text{sta}, k} = \sum_{j=1}^k v_j \Delta\text{Fo}, m_{\text{sta}, k} = \frac{\text{St}_{\text{sta}, k}}{\Delta\text{St}};$$

$$m_{\text{end}, k} = 0 \text{ for } \text{St}_{\text{sta}, k} \leq L_0;$$

$$m_{\text{end}, k} = m_{\text{sta}, k} - \frac{L_0}{\Delta\text{St}} \text{ for } \text{St}_{\text{sta}, k} > L_0.$$

As a result of the finite-difference approximation of Eqs. (17) and (21) we obtain calculating equations for the temperatures of the gas and the substance along the length of the interval $m_{\text{end}, k} < m < m_{\text{sta}, k}$ of the layer:

$$t(r)_{m+1, k} = T_{m+1, k-1} - \sum_{n=1}^{\infty} A(r, \mu_n) [T(0, \text{St}_{m+1}) \exp(-\mu_n^2 \text{Fo}_{m+1, k}) + h_{m+1, k-1, n}] + \sum_{n=1}^{\infty} A'(r, \mu_n, t_{m+1}^0) \exp(-\mu_n^2 \text{Fo}_{m+1, k}); \quad (24)$$

$$T_{m+1, k} = \frac{T_{m, k} z_{m, k} + \Delta\text{St} [t_{\text{sur}, m+1, k} - \kappa'(T_{m, k}) + Q'_{m, k}]}{z_{m, k} + \Delta\text{St}}, \quad (25)$$

$$\text{where } h_{m+1, k, n} = h_{m+1, k-1, n} \exp(-\mu_n^2 \Delta\text{Fo}) + T_{m+1, k} - T_{m+1, k-1};$$

$$\text{for } \text{Fo}_{m+1, k} = 0 \quad h_{m+1, k, n} = 0.$$

Here and later the indices correspond to counterflow. For direct flow the index $m+1$ is changed to $m-1$. $T(0, \text{St}_{m+1})$, the gas temperature at the moment of loading the $m+1$ -th element of the layer into the oven, is determined from (25). If one takes $\text{Fo}_{m+1, k} = 0$ then $T_{m+1, k} = T(0, \text{St}_{m+1})$, hence it follows that

$$T(0, \text{St}_{m+1}) = \frac{T_{m, k} z_{m, k} + \Delta\text{St} [t_{\text{sur}, m+1}^0 - \kappa'(T_{m, k}) + Q'_{m, k}]}{z_{m, k} + \Delta\text{St}}. \quad (26)$$

In the case of direct flow $T(0, \text{St}_{m+1}) = T_{\text{in}, k}$ for all elements of the layer. The average-mass temperature of the body can be found from Eq. (24) by replacing the coefficients A and A' in it by B and B' , respectively. In the calculation of the heating of preparation in a continuous counterflow oven with multizone heating (see Fig. 1) one must take in Eqs. (24)-(26)

$$z_{m, k} = \sum_{i=0}^p z_{i, k} \varphi(\text{St}_m - \text{St}_{\text{sta}, k} + L_i);$$

$$Q'_{m, k} = \frac{T_{\text{in}, k} - T_{m, k}}{L_0 \Delta\text{St}} \sum_{i=1}^p z_{i, k} [\varphi(\text{St}_m - \text{St}_{\text{sta}, k} + L_i) - \varphi(\text{St}_m - \Delta\text{St} - \text{St}_{\text{sta}, k} + L_i)];$$

$$A(r, \mu_n) = \frac{2 \sin \mu_n \cos(\mu_n r)}{\mu_n + \sin \mu_n \cos \mu_n}, \quad \mu_n = \text{ctg} \mu_n \text{Bi};$$

$$A'(r, \mu_n, t_{m+1}^0) = \frac{2 \mu_n \cos(\mu_n r)}{\mu_n + \sin \mu_n \cos \mu_n} \int_0^1 t^0(r)_{m+1} \cos(\mu_n r) dr;$$

$$B = \frac{2 \sin^2 \mu_n}{\mu_n (\mu_n + \sin \mu_n \cos \mu_n)}, \quad B' = \frac{2 \sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n} \int_0^1 t^0(r)_{m+1} \cos(\mu_n r) dr.$$

Equations (24)–(26) remain unchanged in application to a stationary layer. Since the oven must be filled with the substance from the very start of the heating, for all the elements of the stationary layer within the oven $\tau_{d, m+1} = 0$ and $Fo_{m+1, k} = Fo_k = k\Delta Fo$. The numerical–analytical method presented for the solution of the general problem of nonsteady-layer heat exchange can be applied to the heating of a layer by radiation. For this we introduce a correction for the radiant heat flux in Eqs. (17) and (18):

$$z(Fo', St) \frac{\partial T}{\partial St} = t(1, Fo, St) - T'(Fo, St) - \kappa'(T) + Q'(Fo', St); \quad (17')$$

$$\left. \frac{\partial t}{\partial r} \right|_{r=1} = Bi [T'(Fo, St) - t(1, Fo, St)], \quad (18')$$

where

$$T'(Fo, St) = t(1, Fo, St) + \frac{\sigma t_{cal}^3}{\alpha} [T^4(Fo, St) - t^4(1, Fo, St)],$$

$$t = \frac{t_s}{t_{cal}}, \quad T = \frac{t_g}{t_{cal}}, \quad \alpha = \text{const} \neq 0. \quad (27)$$

The value of α is chosen arbitrarily. In the calculating equation (24) the values denoted by the letter T are changed respectively to T'. The gas temperature is determined from Eq. (17'), which after the finite-difference approximation takes the form

$$T_{m+1, k} + \frac{\Delta St}{z_{m, k}} \cdot \frac{\sigma t_{cal}^3}{\alpha} T_{m+1, k}^4 = T_{m, k}$$

$$- \frac{\Delta St}{z_{m, k}} \left[- \frac{\sigma t_{cal}^3}{\alpha} t_{sur, m+1, k}^4 + \kappa'(T_{m, k}) - Q'_{m, k} \right]. \quad (28)$$

On the basis of the equations obtained an algol-program was developed and calculations were conducted on a Minsk-22 computer for the temperature fields of the gas and substance during nonsteady heat exchange by convection and radiation in application to a continuous oven.

NOTATION

t_s, t_g : temperature of substance and gas; t_s^0, t_g^0 : same at the start of heating; \bar{t} : average mass temperature of body; t_{cal} : calorimetric temperature of fuel combustion; $\nu = 0, 1, 2$ for a plate, cylinder, and sphere, respectively; R: thickness of bodies comprising the layer; y: coordinate of a point through the thickness of the body; τ : heating time of material; a : thermal diffusivity; λ : thermal conductivity; ρ : density; α : heat exchange coefficient; σ : radiation coefficient; c_g : specific heat capacity of gas; V_g : current flow rate of gas; V_{max} : flow rate of gas at maximum thermal power of oven; $V_S c_S$: water equivalent of substance; κ : coefficient of heat transfer from gas to surrounding medium; κ' : function of heat loss to surrounding medium; f_{1i} : surface of oven lining per unit mass of substance; q: power of heat source; q': power of heat source per unit heating surface; p: number of zones of oven; i: ordinal number of zone; Fo_i : dimensionless time of movement of body from oven entrance to end of i-th zone; T_{in} : gas temperature at oven entrance; V_{gi} : flow rate of gas through burners of i-th zone; St_{sta}, St_{end} : current coordinates of start and end of oven along length of layer; L_0 : dimensionless length of oven; f : heating surface from entrance of layer to any cross section; f_{act} : area of active oven hearth; k: ordinal number of time interval; m: ordinal number of element along length of layer; φ : unit function; t_{sur} : surface temperature of body; $\tau_{d, m+1}$: delay time for heating of m + 1-th element of layer.

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